Proseminar: Quadratic forms

Prof. Dr. Gebhard Böckle **WS 2023**

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Timings(Tentative): Mondays, 2 PM-4 PM; INF 205, Room: SR8

Target Group: Bachelor students

Prerequisites: Knowledge of Linear Algebra 1-2 is expected. Some background in Analysis is appreciated.

Requirements: Talks are supposed to be in English. For each talk, duration is 90 minutes and a handout is expected.

Brief description: Let k be a field (characteristic $\neq 2$). By a *quadratic form* $\mathbf{f}(\mathbf{x})$ over k in n variables $\mathbf{x} = (x_1, ..., x_n)$ we mean a function of the form

$$\mathbf{f}(\mathbf{x}) = \sum_{i,j} f_{ij} x_i x_j = f_{11} x_1^2 + 2 f_{12} x_1 x_2 + ... (f_{ij} = f_{ji})$$

Suppose you are given a quadratic form $\mathbf{f}(\underline{x})$ with $f_{ij} \in \mathbb{Z}$ and asked to find an integer solution. The most natural thing you would do is to check whether $\mathbf{f}$ has a solution in each $\mathbb{Z}/m\mathbb{Z}$ and also in $\mathbb{R}$. For example, $x^2 + y^2 = -z^2$ has no integer solutions due non-existence of real roots. Similarly $x^2 + y^2 = 6z^2$ has no integer solutions due to non-existence of solutions of the equation in $\mathbb{Z}/3\mathbb{Z}$. Conversely, it is natural to wonder if $\mathbf{f}$ has an integer solution, given that it has solutions in all $\mathbb{Z}/m\mathbb{Z}(m \in \mathbb{N}_{>0})$ and in $\mathbb{R}$.
The answer is yes(for regular forms) and this is called the *Hasse principle*[1].
In this proseminar, we aim at formulating the Hasse principle, in terms of notions which are tractable, and then study a proof of it. The primary reference for this proseminar is the book *Rational quadratic forms* by J. W. S. Cassels. Successful participation in the proseminar will help the student to explore more advanced texts on quadratic forms.

Vorbesprechung: (Tuesday)18.07.23, 11:30 a.m.(s.t.) in SR8, INF 205

Seminar homepage: https://typo.iwr.uni-heidelberg.de/groups/arith-geom/members/sriramcv/quadratic-forms

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[1] Sometimes also called the *Hasse-Minkowski principle*

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